Monday, April 16, 2012
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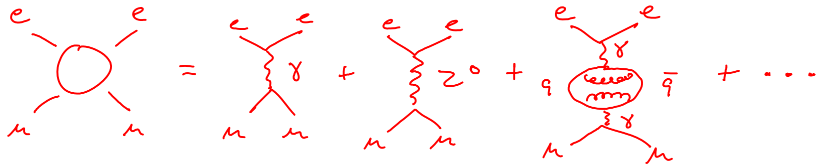
Heirarchy of interactivity:

neutrinos - Weak
charged leptons - E+M, Weak
quarks - QCD, E+M, Weak

"direct" interactions but...

In a given process M must include all contributions!



E+M   Weak   E+M + QCD

Virtual states can bring in additional interactions!

Our next step will be to discuss QCD which means quarks, but we must also understand how QED plays out for quarks. We'll save the Weak interactions for last!

Quarks + QED

Quarks can replace $e^\pm, \mu^\pm, \tau^\pm$ in any QED diagram w/ appropriate charge of $g_e = e\sqrt{\frac{4\pi}{\hbar c}}$ (vertex factors).

$q = \frac{1}{3}e$ or $\frac{2}{3}e$

Everything else same: $\underbrace{u, v, \bar{u}, \bar{v}}_{\text{spin-}\frac{1}{2}\text{ quarks}}, \underbrace{\epsilon_\mu, \epsilon^*_\mu}_{\gamma\text{'s}}$ external states

propagators same, etc. (Note: color just along for ride)

Quarks + QCD

Color $\underbrace{(r, g, b)}_{}$ plays role of "charges" and interactions via $\underbrace{8\text{ gluons}}_{}$.

3 charges instead of 1 in QED! ⟵ Note in QED only <u>one</u> γ!

One practical complication is that QCD is defined in terms of quarks, but we only observe (and experiment w/) hadrons.

In fact since we often work with mesons $(q\bar{q})$ and gluons $(c\bar{c})$, it is useful to work with a linearly independent set of $c\bar{c}$ basis states:

"Octet"

$$\underbrace{\begin{aligned}
|1\rangle &= \tfrac{1}{\sqrt{2}}(r\bar{b} + b\bar{r}) & |5\rangle &= -\tfrac{i}{\sqrt{2}}(r\bar{g} - g\bar{r}) \\
|2\rangle &= -\tfrac{i}{\sqrt{2}}(r\bar{b} - b\bar{r}) & |6\rangle &= \tfrac{1}{\sqrt{2}}(b\bar{g} + g\bar{b}) \\
|3\rangle &= \tfrac{1}{\sqrt{2}}(r\bar{r} - b\bar{b}) & |7\rangle &= -\tfrac{i}{\sqrt{2}}(b\bar{g} - g\bar{b}) \\
|4\rangle &= \tfrac{1}{\sqrt{2}}(r\bar{g} + g\bar{r}) & |8\rangle &= \tfrac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g})
\end{aligned}}_{\text{Rotated into each other under } SU(3)}$$

Note: $|3\rangle, |8\rangle$ are "colorless" but nontrivial!

"Singlet" $\underbrace{|9\rangle = \tfrac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})}_{\text{Invariant under } SU(3)}$

Quarks must exist in singlet combinations (for mesons).
Gluons only appear in Octet combinations. If a $|9\rangle$ gluon existed it could exist on its own and would behave much like γ. But we have never seen such a particle! Also w/ $|9\rangle$ $\underset{8\text{-generators}}{SU(3)} \rightarrow \underset{9\text{-generators}}{U(3)}$.

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Now that we have the field strength $F_{\mu\nu}$, we add the gauge invariant term:

$$\mathcal{L}_{A_\mu} = \frac{1}{16\pi} F_{\mu\nu}^{a} F^{\mu\nu a} = \frac{1}{16\pi} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} - g f^{ade} A^{\mu d} A^{\nu e})$$

$$= \underbrace{\frac{1}{16\pi} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a})}_{\text{usual kinetic term}} \left. \begin{array}{l} - \frac{g}{16\pi} f^{ade} A_\mu^d A_\nu^e (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) \\ - \frac{g}{16\pi} f^{abc} A^{\mu b} A^{\nu c} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \\ + \frac{g^2}{16\pi} f^{abc} f^{ade} A_\mu^b A_\nu^c A^{\mu d} A^{\nu e} \end{array} \right\} \text{These are gluon-gluon interactions!}$$

Note that the gluon-gluon interactions critically depend on $SU(3)$ being non-abelian, i.e. $f^{abc} \neq 0$. This is of course why photons in (abelian $U(1)$) E&M do not interact w/ each other (at least classically).

These gluon-gluon interactions bring in a host of new effects including glueballs, confinement, etc.

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A useful way to think about quark gluon interactions is as follows.

Consider $\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$ and A_μ^1 associated w/ $\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ [again from HW3]

Then: $\bar{\psi} \lambda \cdot A_\mu^1 \psi = (\bar{\psi}_r \, \bar{\psi}_b \, \bar{\psi}_g) \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\frac{1}{\sqrt{2}}(\underbrace{b\bar{r}}_{b\bar{r}(r)=b} + \underbrace{r\bar{b}}_{r\bar{b}(b)=r})} \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix} = \bar{\psi}_r \psi_b + \bar{\psi}_b \psi_r + \bar{\psi}_g \psi_g$

So the gluons are bi-colored ($c\bar{c}$) while the quarks are just colored (r, b, g), anti-quarks $(\bar{r}, \bar{b}, \bar{g})$.
This will be immensely helpful in constructing Feynman diagrams for QCD.

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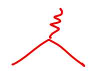
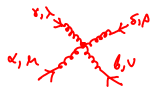
Let's compare:

	External State Labels		Internal Propagators	Vertex Factors	
ABC	none		$\frac{i}{q^2 - m^2c^2}$		$-ig$
QED	$u, \bar{u}, v\ \bar{v}$	"matter"	$\frac{i(\not{q} + mc)}{q^2 - m^2c^2}$		$ig_e\gamma^\mu$
	$\epsilon_\mu, \epsilon^*_\mu$	"photons"	$-\frac{ig_{\mu\nu}}{q^2}$		
QCD	$uc, \bar{u}c^\dagger, vc, \bar{v}c^\dagger$	"matter"	$\frac{i(\not{q}+mc)}{q^2 - m^2c^2}$		$-\frac{ig_s}{2}\lambda^\alpha\gamma^\mu$
	$\epsilon_\mu a^\alpha;\ \epsilon^*_\mu a^{\alpha *}$	"gluons"	$-\frac{ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$		$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1-k_2)_\lambda + g_{\nu\lambda}(k_2-k_3)_\mu + g_{\lambda\mu}(k_3-k_1)_\nu]$
					$-ig_s^2[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]$.

Okay, so what are $c, a^\alpha, \lambda^\alpha, f^{\alpha\beta\eta}$?

c: There are 3 colors so "charge space" is 3D w/

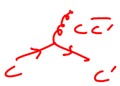
$c_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} = \text{red}$ $\quad c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} = \text{blue}$ $\quad c_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} = \text{green}$ basis in H^3 space of color states.

An arbitrary state can have $c = a_i c_i$

$\quad$ ↳ complex coefficients, hence $c^\dagger = c^{T*}$

a^α: There are 8 gluons w/

$a^1 = \begin{pmatrix}1\\0\\ \vdots\end{pmatrix}$ $\quad a^2 = \begin{pmatrix}0\\1\\0\\ \vdots\end{pmatrix}$ $\quad \cdots \quad a^8 = \begin{pmatrix}\vdots\\0\\1\end{pmatrix}$ basis in H^8 space of gluon states.

Recall  ⟵ naively $3\times 3 = 9$ gluons, but the theory only includes 8 transformed into each other by $SU(3)$ (which has 8 generators!)

The "singlet" gluon $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$ does not exist (would look a lot like γ, and we haven't seen it!), Of course we haven't seen the other gluons, but for those we can use the confinement argument.

If the singlet did exist, then we would say QCD ⟷ $U(3)$ (instead of $SU(3)$).

λ^α: For QED the γ^μ_{ab} matrices linked "spin-space" to "space-time", i.e. 4 4×4 matrices.

The λ^α_{ij} matrices of QCD link H^3 of "color-space" to H^8 of "gluon-space", i.e. 8 3×3 matrices (8.34).

Just like w/ γ^μ, we will leave off color-space labels and write λ^α (and c instead of c_i).

$f^{\alpha\beta\eta}$: $[\lambda^\alpha, \lambda^\beta] = 2i\underbrace{f^{\alpha\beta\gamma}}\lambda^\gamma$ 512 structure constants of $SU(3)$ Lie Algebra

One huge technical complication that arises is for diagrams involving internal loops. In this case we have to be very careful not to count gauge equivalent (physically indistinct) configurations more than once. To get the counting right w/out losing gauge invariance, we introduce Faddeev-Popov ghosts which are additional nonphysical fields whose sole purpose is to cancel the nonphysical gauge equivalent fluctuations of the physical fields.